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Enhanced Andreev scattering by order–disorder transitions in short coherence length superconductors

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Abstract. We predict that the excitations of a superconductor with a spatially inhomogeneous order parameter $\Delta(\boldsymbol{r})$ can be localised by Andreev scatering. Results of calculations on two models are presented. The first contains fluctuations only in the magnitude of $\Delta(\boldsymbol{r})$, while the second has fluctuations only in the phase of $\Delta(\boldsymbol{r})$. For both models, our results suggest that all quasi-particle states are localised in one dimension. In particular we show that for quasi-particles incident on the superconductor from a normal wire, the reflection coefficient for Andreev scattering can approach unity, even for energies far above the maximum value of $|\Delta(\boldsymbol{r})|$. This leads to new charging effects which can be used as an experimental probe for order–disorder transitions in $\Delta(\boldsymbol{r})$.

The existence of short coherence lengths ξ in high T_c superconductors, leads to new properties that are absent in more conventional superconductors. For example in the former, the melting curve for an Abrikosov flux lattice [1–4] is quite distinct from the boundary between the normal and mixed states, whereas in conventional type-II superconductors these are practically indistinguishable. Such changes in the scale of spatial variation of the order parameter $\Delta(\boldsymbol{r})$ are expected to yield corresponding changes in the excitations of the system. Whereas quasi-classical arguments [5] yield excellent results when ξ, in units of inverse Fermi wavevector, is of order 10^3, a more accurate solution of the Bogoliubov–de Gennes (BG) equation is needed when ξ is of one or two orders of magnitude smaller.

In this paper, we predict a new consquence of the smallness of ξ, namely the localisation of quasi-particles due to particle–hole scattering by a spatially inhomogeneous order parameter. This mechanism for localisation is distinct from that of Anderson localisation, which arises from elastic scattering in disordered *normal* solids [6]. Such scattering contributes only to diagonal terms in the BG equation. In what follows it is shown that particle–hole scattering alone is sufficient to localise electrons. We also show that apart from more obvious consequences for transport properties, this phenomenon can lead to new charging effects which constitute a novel probe into order–disorder transitions of Δ $(\boldsymbol{r})$.

To demonstrate localisation by particle–hole scattering, we examine the simplest model one can envisage, based on the BG equation in one dimension

$$\begin{pmatrix} -\partial_x^2 - 1 & \Delta(x) \\ \Delta^*(x) & \partial_x^2 + 1 \end{pmatrix} \begin{pmatrix} \psi(x) \\ \varphi(x) \end{pmatrix} = E \begin{pmatrix} \psi(x) \\ \varphi(x) \end{pmatrix}. \tag{1}$$

Here all energies are measured in units of the chemical potential $\mu_S = \hbar^2 k_F^2/2m$ and all

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lengths in units of k_F^{-1}. In what follows, we regard $\Delta(x)$ as given, at least in a probabilistic sense. Simple models based on the BG equation with a *specified* $\Delta(x)$ have proved to be particularly useful for understanding the properties of normal–superconducting–normal (N–S–N) [7], S–N–S [8] and N–S [9–11] junctions. In the latter case, if $\Delta(\boldsymbol{r})$ approaches a constant value Δ_0 in the bulk of the superconductor, the dominant scattering mechanism for an incoming electron from the left (of energy $E < \Delta_0$) is its conversion to an outgoing hole on the left, accompanied by the emission of a Cooper pair into the condensate. Since the Cooper pair carries charge, but not entropy, such off-diagonal, Andreev scattering has a marked effect on the thermal conductance, but the electrical conductance is almost unchanged. On the other hand, for $E/\Delta_0 > 1$ such studies show that the transmission coefficient for quasi-particles rapidly approaches unity. We now demonstrate that if $\Delta(x)$ is spatially disordered within the bulk of the superconductor, this result is drastically changed.

To this end, imagine dividing the superconductor, whose ends are attached to normal leads, into cells of length ξ and take the order parameter within cell j to be a constant of the form $\Delta_j\,e^{i\theta_j}$. Two models will be considered, the first of which is invariant under time reversal, while the second is not. In model 1, $\theta_j = 0$ for all j, but Δ_j is a random number chosen from a uniform distribution in the interval $\Delta_0(1 \pm \delta_\Delta)$. In model 2, $\Delta_j = \Delta_0$ for all j, whereas θ_j is a random number chosen from a uniform distribution in the range $\pm\delta_\theta$. For a superconductor with no disorder, one expects $\xi \sim \Delta_0^{-1}$ and in what follows the choice $\xi^{-1} = \Delta_0 = 10$ is made.

To calculate the reflection and transmission coefficients, the solution of equation (1) in cell j is written

$$\begin{pmatrix}\psi(x)\\ \varphi(x)\end{pmatrix} = \begin{pmatrix}U_{k_j}\\ V_{k_j}\end{pmatrix}(A_{\rm p}^{(j)}\,e^{ik_jx} + B_{\rm p}^{(j)}\,e^{-ik_jx}) + \begin{pmatrix}U_{q_j}\\ V_{q_j}\end{pmatrix}(A_{\rm h}^{(j)}\,e^{iq_jx} + B_{\rm h}^{(j)}\,e^{-iq_jx})$$

where $U_{k,q}$ and $V_{k,q}$ are the usual BCS amplitudes for cell j, satisfying

$$|U_{k,q}|^2 + |V_{k,q}|^2 = 1$$

and

$$U_k/V_k = (V_q/U_q)^* = (\Delta_j\,e^{i\theta_j})/[E - (k^2 - 1)].$$

The quantities $\pm k$ and $\pm q$ are the wavevectors of the degenerate particle-like and hole-like channels respectively. The condition that the solution of (1) and its first derivative must be continuous at a cell boundary, yields a collection of 4×4 transfer matrices relating the plane wave amplitudes of successive cells, whose product is the transfer matrix $\mathbf{T}$, connecting the amplitudes $A_{\rm p,h}^{(0)}$ and $B_{\rm p,h}^{(0)}$ of the normal wire on the left of the superconductor to those of the normal wire on the right.

Given $\mathbf{T}$, it is a straightforward matter to construct the corresponding $\mathbf{S}$-matrix connecting incoming to outgoing channels and from this a matrix $\mathbf{P}$ of reflection and transmission coefficients for fluxes. If we associate subscripts p, h (p′, h′) with particle and hole fluxes on the left (right) then $\mathbf{P}$ is of the form

$$\mathbf{P} = \begin{pmatrix} R_{\rm pp} & R_{\rm ph} & T_{\rm pp'} & T_{\rm ph'} \\ R_{\rm hp} & R_{\rm hh} & T_{\rm hp'} & T_{\rm hh'} \\ T_{\rm p'p} & T_{\rm p'h} & R_{\rm p'p'} & R_{\rm p'h'} \\ T_{\rm h'p} & T_{\rm h'h} & R_{\rm h'p'} & R_{\rm h'h'} \end{pmatrix}.$$

If a unit particle (hole) flux is incident from the left, then the first and second elements

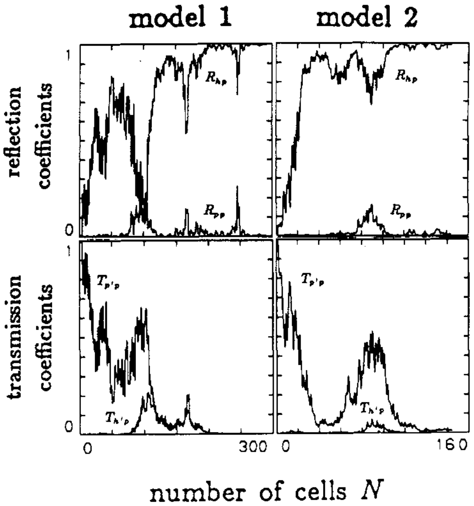

Figure 1. Reflection and transmission coefficients as functions of the system size N, for $E = 4\Delta_0$, $\delta_\Delta = 1$ (model 1) and $\delta_\theta = \pi/2$ (model 2).

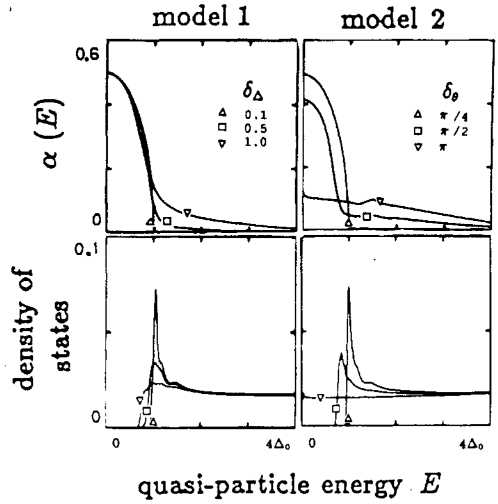

Figure 2. Plots of the inverse localisation length $\alpha(E)$ and density of states $N(E)$ versus energy E for disorders of $\delta_\Delta = 0.1, 0.5, 1.0$ (model 1) and $\delta_\theta = \pi/4, \pi/2, \pi$ (model 2).

of the first (second) column of **P** are equal to the reflected particle and hole fluxes respectively and the third and fourth elements are respectively equal to the transmitted particle and hole fluxes. Similarly if a unit particle (hole) flux is incident from the right, then the elements of the third (fourth) column of **P** are equal to the transmitted particle and hole fluxes and the reflected particle and hole fluxes respectively. In general, the 16 elements of **P** are distinct, satisfying only the requirement of quasi-particle number conservation, namely

$$\sum_{i=1}^{4} P_{ij} = 1$$

for all j. However, in the presence of time reversal symmetry as in model 1, it is easy to show that **P** is symmetric. Furthermore, for equal particle and hole wavevectors ($k = q$), **P** is invariant under an interchange of indices $\mathrm{p} \leftrightarrow \mathrm{h}$ and $\mathrm{p}' \leftrightarrow \mathrm{h}'$. For $k \simeq q$ this is a useful approximate symmetry. Finally, although inversion symmetry is broken for a given sample, it may be present on average. In this case, the ensemble averaged matrix $\langle \mathbf{P} \rangle$, is invariant under an interchange of primed and unprimed subscripts. In what follows, we deal with *non-averaged* quantities only.

Figure 1 shows typical results for the variation of the reflection/transmission coefficients with the size (i.e. the number of cells N) of the superconductor. For model 1, the values $E = 4\Delta_0$, $\delta_\Delta = 1$ were used and for model 2, $E = 4\Delta_0$, $\delta_\theta = \pi/2$. In addition to these values, we have obtained results for a selection of E in the range $0 < E < 4\Delta_0$ and disorders in the range $0 < \delta_\Delta < 1$, $0 < \delta_\theta < \pi$. In common with figure 1, these results strongly suggest that in the limit of large N, whatever the value of E, δ_θ and δ_Δ, all transmission coefficients vanish. More important they reveal that in this limit, the *normal* reflection coefficients R_{pp}, R_{hh}, $R_{\mathrm{p'p'}}$, $R_{\mathrm{h'h'}}$ also vanish and consequently an incoming quasi-particle of arbitrary energy is guaranteed to be Andreev reflected.

As a measure of the degree of localisation, we extract the four Lyapanov exponents of **T**, which occur in +, − pairs and plot the smallest positive exponent $\alpha(E)$ as a function of E. Following well known treatments of multi-channel scattering [12–14], one expects the transmission coefficients to decay exponentially with an inverse decay length $\alpha(E)$, which is therefore identified with the inverse localisation length for quasi-particles of energy E. As one might expect from known properties of products of random matrices [13, 14], $\alpha(E)$ self-averages in the large-N limit and therefore no ensemble averaging is required. The results shown below were obtained from calculations on $N = 10^6$ cells. For both models 1 and 2, figure 2 shows numerical results for $\alpha(E)$ along with results for the average density of quasi-particle states per unit length $N(E)$, obtained using a node counting technique. These suggest that for any non-vanishing disorder, all states are localised. It is interesting that these results are qualitatively of the form obtained for disordered normal systems [15] suggesting that an analysis based on phase recurrence relations may be a useful approach to computing $\alpha(E)$ analytically.

For large E, $\alpha(E)$ decreases monotonically. Indeed our results suggest that for model 1, $\alpha(E) \sim E^{-1/2}$ and for model 2, $\alpha(E) \sim E^{-3/2}$. This is illustrated in figure 3, which shows that log–log plots of the quantities $\alpha_1(E) = E^{1/2}\alpha(E)$ and $\alpha_2(E) = E^{3/2}\alpha(E)$ respectively, versus disorder, fall onto single scaling curves, independent of E. Furthermore, since the curves approach straight lines at small disorder, one finds power law behaviours of the form $\alpha(E) \sim (\delta_\Delta)^{\sigma_1}$ and $\alpha(E) \sim (\delta_\theta)^{\sigma_2}$ for models 1 and 2 respectively. From the slopes of the straight lines in this figure, one obtains $\sigma_1 = 2.02 \pm 0.02$ and $\sigma_2 = 1.99 \pm 0.01$, which are close to the value [16] of $\sigma = 2$ obtained for the Anderson model of disordered normal solids in one dimension. Thus, our results suggest that in the limit of large E and small disorder, $\alpha(E) \sim E^{-1/2}\,(\delta_\Delta)^2$ and $\alpha(E) \sim E^{-3/2}(\delta_\theta)^2$ for models 1 and 2 respectively.

The results in figures 1–3 demonstrate for the first time the occurrence of localisation due to particle–hole scattering in an inhomogeneous superconductor. As noted earlier, for a normal metal in contact with a homogeneous superconductor the probability R_{hp} of Andreev reflection rapidly approaches zero for $E > \Delta_0$. For an inhomogeneous superconductor of length L, this is no longer valid and must be replaced by the condition $\alpha(E)L \ll 1$. Consequently, quasi-particles with energies much larger than Δ_0 can be Andreev reflected with a high probability. This immediately leads to the possibility of detecting order–disorder transitions, such as the melting of a flux lattice [1, 2], by monitoring changes in transport properties. For example, one expects such a transition to be accompanied by a sharp drop in the electronic contributrion to the thermal conductance.

A more novel effect accompanying an order–disorder transition arises from the non-conservation of quasi-particle charge associated with Andreev processes. Since an Andreev-scattered particle (hole) changes the charge on the superconductor by an amount $+\,(-)\,2e$, any change in the reflection coefficients for this process will cause the superconductor to adjust its chemical potential μ_{S} in order to re-establish equilibrium. To illustrate this and to obtain an estimate of the size of the effect consider, as illustrated in figure 4, two reservoirs at chemical potentials $\mu_1 > \mu_{\mathrm{S}}$ and $\mu_2 < \mu_{\mathrm{S}}$ connected via oxide layers to the ends of a long superconducting wire. The oxide layers ensure that at equililbrium there is no chemical potential difference across the superconductor and, by varying their relative thicknesses, they can be used to control the relative numbers of quasi-particles from the left and right. At zero temperature, the particle branch in the left reservoir is populated from μ_{S} to $\mu_{\mathrm{S}} + (\mu_1 - \mu_{\mathrm{S}}) = \mu_{\mathrm{S}} + \bar{\mu}_1$ and the hole branch in the right reservoir from μ_{S} to $\mu_{\mathrm{S}} + (\mu_{\mathrm{S}} - \mu_2) = \mu_{\mathrm{S}} + \bar{\mu}_2$.

Let the number of particles (holes) per unit energy per unit time impinging on the superconductor from the left (right) be n_1 (n_2). Then assuming for convenience that n_1

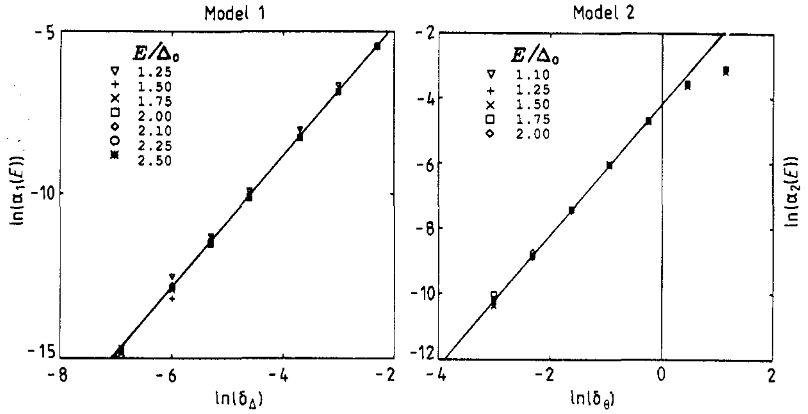

Figure 3. Plots of the scaled inverse localisation lengths $\alpha_1(E) = E^{1/2}\alpha(E)$ and $\alpha_2(E) = E^{3/2}\alpha(E)$ for models 1 and 2 respectively, as functions of the disorder. The full lines are straight lines of slope 2.

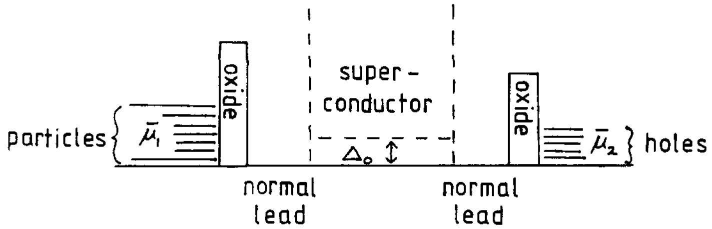

Figure 4. Schematic diagram showing the situation arising at zero temperature when reservoirs at different chemical potentials μ_1 and μ_2 are attached via normal leads and oxide layers of different thicknesses to opposite ends of an inhomogeneous superconductor.

and n_2 are energy independent over the energy range of interest, the currents I_1 and I_2 in leads 1 and 2 respectively are

$$I_1 = n_1 \int_{\mu_S}^{\mu_S+\bar{\mu}_1} dE\,(1 - R_{pp} + R_{hp}) + n_2 \int_{\mu_S}^{\mu_S+\bar{\mu}_2} dE\,(T_{hh'} - T_{ph'})$$

$$I_2 = n_1 \int_{S}^{\mu_S+\bar{\mu}_1} dE\,(T_{p'p} - T_{h'p}) + n_2 \int_{\mu_S}^{\mu_S+\bar{\mu}_2} dE\,(1 + R_{p'h'} - R_{h'h'}).$$

To obtain an estimate of the size of the charging effect we make the following simplifying assumptions. For the ordered case: when $E < \Delta_0$, $R_{hp} = R_{ph} = R_{h'p'} = R_{p'h'} = 1$ and all other coefficients vanish and when $E > \Delta_0$, $T_{p'p} = T_{h'h} = T_{h'h'} = T_{pp'} = 1$ and all other coefficients vanish. For the disordered case; a size dependent mobility edge $\bar{\Delta}_0 > \Delta_0$ exists, such that for $E < \bar{\Delta}_0$, $R_{hp} = R_{ph} = R_{h'p'} = R_{p'h'} = 1$ and all other coefficients vanish, while for $E > \bar{\Delta}_0$, $T_{p'p} = T_{h'h} = T_{hh'} = T_{pp'} = 1$ and all others vanish. Since, in practice, $n_1 - n_2$ will never be precisely zero, we assume in what follows, $n_1 < n_2$, which implies that at equilibrium $\bar{\mu}_2 < \Delta_0$ and $\bar{\mu}_2 < \bar{\Delta}_0$ in the ordered and disordered cases respectively. Furthermore, if $\bar{\mu}_1 < \Delta_0$, μ_S is unaffected by an order–disorder transition. For this reason, the discussion is restricted to the case $\bar{\mu}_2 < \Delta_0$, $\bar{\mu}_1 > \Delta_0$. This yields, for the ordered case, $I_1 = n_1(\bar{\mu}_1 + \Delta_0)$, $I_2 = n_1(\bar{\mu}_1 - \Delta_0) + 2n_2\bar{\mu}_2$ which, after setting $I_1 = I_2$ yields $\bar{\mu}_2 = (n_1/n_2)\,\Delta_0$. Similarly in the disordered case, if χ denotes the smaller of $\bar{\Delta}_0$ and $\bar{\mu}_1$, one finds $\bar{\mu}_2 = (n_1/n_2)\,\chi$. These equations are trivially rearranged to yield the chemical potential of the superconductor in the ordered and disordered cases. Taking the difference between the two results, shows that the change in μ_S due to this order–disorder transition is $\mu = (\chi - \Delta_0)(n_1/n_2)$. As an example for $n_1 \simeq n_2$ and $\bar{\mu}_1 > \bar{\Delta}_0$, $\delta\mu \simeq \bar{\Delta}_0 - \Delta_0$. This extremely simple result, demonstrates that the energy scale for the charging effect is Δ_0, which is of the order 10^{-2}–10^{-3} eV and therefore readily measurable.

In this paper, we have demonstrated that spatial fluctuations in a superconducting order parameter provide a new mechanism for 'over the barrier' Andreev reflection and consequently can strongly modify quasi-particle transport properties. For simplicity the analysis has been restricted to a one-dimensional structure at zero temperature. At finite temperatures in common with localisation in normal metals, one expects the system size L in our analysis to be replaced by the inelastic quasi-particle scattering length l. Several interesting questions remain to be answered. For example, what are the critical exponents in two and three dimensions and do non-trivial 'mobility edges' exist? What is the interplay between diagonal (i.e. normal) and off-diagonal disorder in the BG equations? With a view to addressing these questions, we have embarked on detailed calculations in two dimensions, and expect to report the results in the near future.

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